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Solution of an Economic Model using Dynamic Deformation Procedure

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ABSTRACT: Since most of the economic problems are related to the optimization problems, therefore the techniques of the Dantzig[1] can be applied, to all such economic problems in which these problems corresponds to linear form under some constraints conditions. Many nonlinear variety of economic problems are solved by the nonlinear techniques based on the quadratic programming method under the Kuhn-Tucker[10] conditions.

Scarf developed a finite algorithm based on the subdivision of the simplex into finite subsets called primitive sets, then utilizing the Lemke complementarity pivoting procedure he provided the first constructive proof to Brouwer's fixed point theorem. Scarf's work attracted many others researchers. Thereafter, many important refinement and extensions to his algorithm were developed with the help of triangulation of simplex carried by Kuhn[7], Todd[21], Eaves [2] and Merril [16] etc.

However, algorithm developed by Freudenthal[5], Kuhn[7,8],Todd [21] etc. were the extension of the Scarf's algorithm to have better approximation of the fixed point of a continuous mapping from a simplex into itself. Whereas Merril[16], Eaves[2], Eaves and Saigal [3] etc. developed several algorithms for points to set mapping or semi continuous mapping.

In this paper we have tried to give a dynamic manifold construction process to find the solution of different type of economic problems.

Keywords: Lower and upper semi continuity; convex hull; affine hull; triangulation; simplex and dynamic manifold.

INTRODUCTION: To find the solution of different types of economic problems, we first set given problems on a unit (n-1)- dimensional simplex $S^{(n-1)}$. An (n-1)-dimensional simplex is defined as the set of all points that can be expressed as convex combination of n affinely independent vectors, in case of the unit simplex these vectors are the unit vectors in n-space. A simplex has faces, which are the (n-2) - simplices formed by dropping each of n vertices in turn. Any simplex can be subdivided so that each of its part is a simplex, called subsimplex and any two sub-simplices share a common boundary of dimension (n-2), if the boundary is face of both of them. This fact underlies the basic pivot step of the algorithm, starting with any sub-simplex of the subdivision of S^{n-1}. If any vertex is dropped then either the remaining vertices span a face lying in the boundary of S^{n-1} or they can be completed by unique vertex to span a new sub-simplex of the subdivision. One method of subdividing the unit sub-simplex is the regular subdivision due to Kuhn[7]. Every vertex of the sub-simplex in a regular subdivision of the unit simplex can be expressed as $(X_1/D, X_2/D, \ldots X_n/D)$ where D is degree of the subdivision, a positive integer, and $X_1, X_2, \ldots X_n$ are non negative integer such that their sum is D. Moreover, these $X_1, X_2, \ldots X_n$ denote vertex of the D-subdivision, in the sense that they are equal to D times the coordinates of some vertex. If x^1 and x^2 are the vertices of the same sub-simplex, then none of their coordinates can differ by more than 1/D. Thus, as D tends to infinity, the distance between any two points in a single such simplex is zero .Precisely the mesh of the subdivision is surely not greater than $\sqrt{n}/D$, which tends to zero as D tends to infinity.

In the classical general equilibrium model of an exchange economy. Three types of agents participate in each economy - consumers, producers, and revenue handling agents. Consumer sell their labour and resource holding, and purchase goods and services in such a way as to maximize satisfaction subject to restriction that expenditure plus tax payments must not exceed endowment income plus revenue transfers. Producers purchase labour and raw materials after tax profits. Revenue handling agents (usually government authorities) collect taxes from producers and consumers and redistribute the revenue among consumer groups. Since real government spend money as well as collect it and give it away, they are often modelled both as consumer and as revenue handling agents.

PRELIMINARIES: By a suitable dynamic interpretation to the deformation procedure, the connecting solution leads to the desired solution. One such suitable deformation method is the dynamic manifold construction procedure. This can be formally summarise as follows:

Definition:

Dynamic Defined Manifolds D_1 and D_2: The dynamic manifold construction procedure is based on a characterization of all possible outcomes of the procedure. Each outcome will be shown to constitute a legitimate pseudo-manifold on S x $[0.\infty)$. The most of the development deals with the triangulations instead of pseudo-manifolds. The two concepts however are equivalent for all practical purposes.

The exposition begins by defining a family D_1 of a triangulations of R^n x $[0.\infty)$ which simultaneously triangulate a subset of R^n x $[0.\infty)$ affinely homeomorphic to S x $[0.\infty)$. The image of D_1 under the homeomorphism yields a family D_2 of a triangulation of S x $[0.\infty)$. Members of the class D_2 represent all possible outcomes of the dynamic construction process.

Define $J_1(\delta, \lambda)$ for $\lambda > 1$ and odd to be the set of all (n+1)- simplices in $J_1(\delta)$ which meet $R^n x(\delta,\delta\lambda)$.

It can be easily shown that $J_1(\delta, \lambda)$ triangulates $R^n x(\delta, \delta\lambda)$. We also know that the triangulation J_3 possesses a hereditary property similar to $J_1(\delta)$.

The section of J_3 which provide manifold blocks of expanding mesh is defined as follows:

Define $J_3(\lambda_1, \lambda_2)$ for integers $\lambda_2 \geq \lambda_1 \geq 0$ to be the set of all (n+1)-simplices in J_3 which meet R^n x $(2^{-\lambda 2}, 2^{-\lambda 1})$.

Define $-J_3(\lambda_1, \lambda_2)$ for integers $\lambda_2 > \lambda_1 \geq 0$ to be the image of $J_3(\lambda_1, \lambda_2)$ under the linear homeomorphism that reverses the sign of the 0-th coordinate of points in R^{n+1} .

The family D_1 of triangulations of R^n x $[0,\infty)$ consists of all collections $\cup B^k$ of (n+1)-simplices taken from sequences $\langle B^k,t^k,\delta^k \rangle$ generated according to the following rules :

k=0: $B^0 = -J_3(0, \lambda)+e_0$ for some integer $\lambda \geq 0$;

$t^0 = 1-2^{-\lambda}$;

$\delta^0 = 2^{-\lambda}$;

k>0 and -odd: $B^k = J_1(\delta^{k-1}, \lambda) + (t^{k-1}-\delta^{k-1})e_0$ for some odd integer $\lambda \geq 3$;

$t^k = t^{k-1} + (\lambda-1)\delta^{k-1}$;

$\delta^k = \delta^{k-1}$;

k>0 and even: Either $B^k = J_3(\lambda,-\log_2\delta^{k-1}) + (t^{k-1}-\delta^{k-1})e_0$ for some nonnegative-

integer $\lambda < -\log_2\delta^{k-1}$;

$t^k = t^{k-1} + (2^{-\lambda}-\delta^{k-1})$;

$\delta^k = 2^{-\lambda}$;

or

$B^k = -J_3(-\log_2\delta^{k-1}, \lambda) + (t^{k-1}-\delta^{k-1})e_0$ for some integer $\lambda > -\log_2\delta^{k-1}$;

$t^k = t^{k-1} + \delta^{k-1}-2^{-\lambda}$;

$\delta^k = 2^{-\lambda}$;

Note that δ^k is always a non-positive integral power of two , so the $\log_2$ terms are always integral. Members of D_1 will be denoted interchangeably by the sequence (B^k,t^k,δ^k) and by $\cup B^k$.

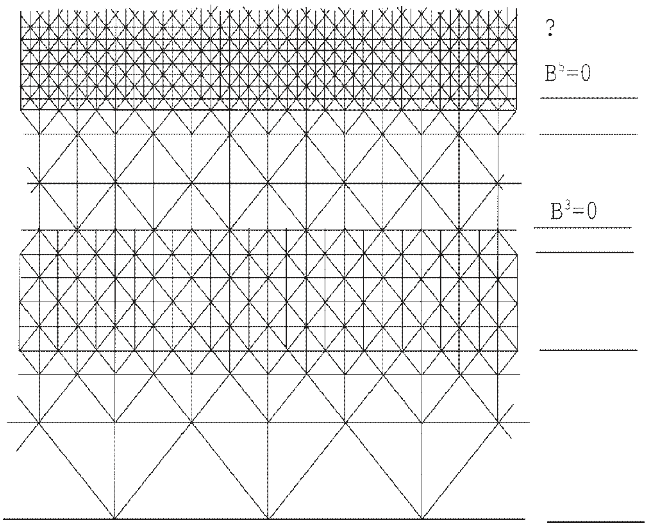

Figure 1: A triangulation of class D_1 on R^1 x $[0,\infty)$.

PROPOSITION: Every member (B^k, t^k, δ^k) of D_1 with $t^k \to \infty$ triangulates R^n x $[0,\infty)$. It will be assumed that all members (B^k, t^k, δ^k) of D_1 satisfy $t^k \to \infty$. The first few blocks of a typical triangulation of class D_2 has been introduced in the Figure1. The right triangles in the figure represent 2-simplices of the triangulation. The sequence (B^k,t^k,δ^k) of block types, block interface heights, and interface scale factors associated with this triangulation is (-1,7/8,1/8), (0,11/8,1/8), (1,3/2,1/4), (0,2,1/4), (-1,35/16,1/16),and (0,?,1/16). (The ? signifies that B^5 is still under construction). The inductive scheme presented in Fig. 1 suggests that each block B^k is defined in entirety at step k of the induction. In practice the block are built-up one layer at a time as the algorithm climbs through $[0,\infty)$. Whenever a new maximum altitude attained, a decision is made whether to extend the present block or begin a new one. In the latter instance the block interface height t^k, the interface scale factor δ^k, and the new block type $ß^{k+1}$ are recorded so that the manifold can be reconstructed should the algorithm turn back down. With these global parameters in place, the block number k and the local representation (y, ψ,a,δ) contain all the information needed to characterize any (n+1)-simplex in a member of D_1.

CONCLUSION: Such dynamic deformation model can be applied for many economic problems to obtain the desired result. Moreover, it can be processed through the computer with suitable algorithm.

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