Asian J. Adv. Basic Sci.: **2015, *3(2)***, 54-55
ISSN (Print): 2454 – 7492
ISSN (Online): 2347 – 4114
www.ajabs.org

Continuity Parameters in Measurable Relations

P. P. Mishra*

Department of Mathematics, Govt. P. G. College, Panna, Madhaya Pradesh, INDIA
** Correspondence: E-mail: mishrapp@yahoo.com*

(Received 11 May, 2015; Accepted 13 May, 2015; Published 13 May, 2015)

ABSTRACT: In this paper we have studied various measurability properties of multifunctions in the context of upper semi continuous mappings. For F_n : T→P_k (x) a multivalued function and T a topological space with Borel σ – algebra of open subsets of T, X a σ - compact metrizable space and P_k (x) a set of compact subsets of x, the measurability and weak measurability have been studied.

Keywords: Measurability; Multifunctions; usc mapings.

INTRODUCTION: The set valued functions which assign to each element t of a measurable space T, a subset of a topological space X in a manner satisfying any one of several possible definitions of measurability are called measurable relations[4]. Various developments in mathematical economics and optimal control have led to the study of the measurability of multi-valued mappings[8].

Himmelberg[4], while studying measurable relations in 1975, considered the general situation where T is an abstract measurable space and X is separable metric. Several authors Aumann[1], Castaing[2], Debreu[3], Jacobs[5], kuratowski and Ryll-Nardzewski[6], McShame and Warfield[7], Rockafellar[8], Van Vleck and Himmelberg[10] have studied extensively different aspects of measurability of multifunctions.

Extending the work of Himmelberg[4], we have considered the case of measurable relation corresponding to upper semi continuous functions. Soni and Mishra[9] have studied the case when the topological space T is measurable under Borel σ - algebra of open sets of T and X is a separable metrizable space. Here our purpose is to investigate the case of weak measurability.

In section 2, we give most of the necessary definitions and terminology, and state without proof some trivial but often required properties of measurable relations in the subsequent section of our main result.

Definitions and some elementary properties:

Throughout the paper, T will denote measurable apace with Borel σ- algebra ß(T) and X is a separable metrizable space.

1. For the mapping F: T→X, if for each closed subset B of X, The set

$$F^{-1}(B) = \{t \in T: F(t) \cap B \neq \emptyset\} \in ß(T)$$

Then the multifunction F is said to be measurable. Also, it is said to be weak measurable, if

$$F^{-1}(G) = \{t \in T: F(t) \cap G \neq \emptyset\} \in ß(T)$$

For any open subset G of X.

2. Let F: T → x be a multifunction, then F is upper semi continuous (u.s.c.) if for each closed subset B of X

$F^{-1}(B) = \{t \in T: F(t) \cap B \neq \emptyset\}$ is closed

And F is lower semi continuous (l.s.c.) if for every open subset G of X

$$F^{-1}(G) = \{t \in T: F(t) \cap G \neq \emptyset\} \text{ is open.}$$

3. Let J be an atmost countable set and let Fn: C→P(X) be a usc relation for each n ε j, where c is compact subset of topological space T and P(X) is the collection of subsets of topological space X, then

i) $\bigcup_{n=1}^{K} F_n$: T→ x is also upper semi continuous

ii) $\prod_{n=1}^{k} F_n$: C→P(X^J) is also upper semi continuous

4. Let J be an atmost countable set and let F_n : T→X be a relation for each n ε j. Then

i) If each F_n is measurable (weak measurable), so is the relation $\bigcup_n F_n$: T→ x defined by$(\bigcup_n F_n)(t) =$ $nFn\ t$; *and*

ii) If X is second countable and each Fn is weakly measurable, then so is the relation, $\prod_n F_n$: T→(X^J) defined by:

$(\prod_n F_n)(t) = \prod_n F_n\ (t)$

Measurability in the presence of upper semi continuity:

We turn now to special criteria for the measurability of multi valued mappings F_n : T→P_k (x) and state the main result as:

Theorem: Let $F_n : T \rightarrow P_k$ (x) be multivalued functions, T be a topological space with Borel σ – algebra of open subsets of T, X be a σ - compact metrizable space and P_k (x) be a set of compact subsets of x. Let each F_n be usc, then

(a) $\bigcup_{n=1}^{K} F_n$: T→ x, defined by $[\bigcup_{n=1}^{K} F_n]$ (t) = $\bigcup_{n=1}^{K} F_n$ (t) is measurable.

(b) $\prod_{n=1}^{k} F_n$: T→(X^J), defined by $[\prod_{n=1}^{k} F_n]$ = $\prod_{n=1}^{k} F_n$ (t) is weakly measurable.

Proof: Applying the compactness of each F_n (t) we get for any closed subset B of X,

F_n^{-1} (B) = T-{t: F_n (t) ⊂ X-B} = T – {t: F_n (t) ⊂ $\bigcup_n A_n$ = T-$\bigcup_n$ {t:F(t) ⊂ A_n }. {Where $A_n = X - B$}

Hence F_n (B) is measurable.

Applying 3(i); F=$\bigcup_{n=1}^{n} F_n$ is also u.s.c. and (a) holds. Further, since each F_n is measurable having compact values, then by hypothesis each F_n will be weakly measurable.

Applying 4(ii), we get:

$\prod_{n=1}^{k} F_n$: T→X^J, defined as $[\prod_{n=1}^{k} F_n]$ (t) = $\prod_{n=1}^{k} F_n$ (t) is weakly measurable.

REFERENCES:

1. Aumann, R. J. (1965) Integrals of Set-Valued Functions, *Journal of Mathematical Analysis and Applications*, 12, 1-12.
2. Castaing, C. (1967) Sur les multi-applications measurables, *Rev. Francais Informat. Researche Operat.*, 1, 91-126.
3. Debreu, G. (1966) Integration of Correspondences, Proc. Fifth. Berkeley Symp. On statistics and Prob.,Vol. II Part I, 351-372 Univ. of California press Berkeley.
4. Himmelberg, C. J. (1975) Measurable relations, *Fundamenta Mathematicae*, 87, 53-72.
5. Jacobs, M. Q. (1968) Measurable multivalued mappings and Lusin's Theorem, *Trans. Amer. Meth. Soci.*, 134, 471-481.
6. Kuratowski, K. and Ryll Nardzewski, C. (1965) A general theorem on selectors, *Bull. Acad. Polon. Sci. Ser. Sci. Math. Astronom. Phys.*, 13, 397-403,
7. Mc Shane, E. J. and warfield, R. B. Jr. (1967) On Filippov's implicit function lemma, *Proc. Amer. Math. Soc.*, 18, 41-47.
8. Rockafellar, R. T. (1969) Measurable dependence of convex sets and functions on parameters, *Journal of mathematical Analysis and Applications*, 28, 4-25.
9. Soni, R. K. and Mishra, P. P. (2010) Measurable relation of usc multifunction, Research Link-77, Vol. –IX (6), 107-108.
10. Van Vleck, F. S. and Himmelberg, C. J. (1969) Some selection theorems for measurable functions, *Can. J. Math*, 21, 394-399.